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Study of the semileptonic decays B → π, D → π and D → K

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Abstract. The semileptonic decay $B \to \pi$ is studied starting from a simple quark model that takes into into account the effect of the B^*-resonance. A novel, multiply subtracted, Omnès dispersion relation has been implemented to extend the predictions of the quark model to all q^2 values accessible in the physical decay. By comparison to the experimental data, we extract $|V_{ub}| = (3.4 \pm 0.2(\exp.) \pm 0.7(\text{theory}))\ 10^{-3}$. As a further test of the model, we have also studied $D \to \pi$ and $D \to K$ decays for which we get good agreement with experiment.

PACS. 12.15.Hh Determination of Kobayashi-Maskawa matrix elements – 11.55.Fv Dispersion relations – 12.39.Jh Nonrelativistic quark model – 13.20.He Decays of bottom mesons

1 Introduction

The exclusive semileptonic decay $B \to \pi l^+ \nu_l$ provides an important alternative to inclusive reactions $B \to X_u l^+ \nu_l$ in the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$.

This reaction has been studied in different approaches like lattice QCD (both in the quenched and unquenched approximations), light-cone sum rules (LCSR) and constituent-quark models (CQM), each of them having a limited range of applicability: LCSR are suitable for describing the low momentum transfer square (q^2) region, while lattice-QCD provides results only in the high-q^2 region. CQM can in principle provide form factors in the whole q^2 range but they are not directly connected to QCD. A combination of different methods seems to be the best strategy.

The use of Watson's theorem for the $B \to \pi l^+ \nu_l$ process allows one to write a dispersion relation for each of the form factors entering in the hadronic matrix element. This procedure leads to the so-called Omnès representation, which can be used to constrain the q^2-dependence of the form factors using the elastic $\pi B \to \pi B$ scattering amplitudes. The problem posed by the unknown $\pi B \to \pi B$ scattering amplitudes at high energies can be dealt with by using a multiply subtracted dispersion relation. The latter will allow for the combination of predictions from various methods in different q^2 regions.

In this work we study the semileptonic $B \to \pi l^+ \nu_l$ decay. The use of a multiply subtracted Omnès representation of the form factors will allow us to use the predictions of LCSR calculations at $q^2 = 0$ in order to extend the results of a simple nonrelativistic constituent-quark model (NRCQM) from its region of applicability, near the zero recoil point, to the whole physically accessible q^2 range. To test our model we shall also study the $D \to \pi$ and $D \to K$ semileptonic decays for which the relevant CKM matrix elements are well known and there is precise experimental data.

2 B → πl$^+\bar{\nu}$

The matrix element for the semileptonic $B^0 \to \pi^- l^+ \nu_l$ decay can be parametrized in terms of two dimensionless form factors,

$$\langle \pi(p_\pi) | V^\mu | B(p_B) \rangle = \left(p_B + p_\pi - q \frac{m_B^2 - m_\pi^2}{q^2} \right)^\mu f^+(q^2) + q^\mu \frac{m_B^2 - m_\pi^2}{q^2} f^0(q^2), \qquad (1)$$

where $q^\mu = p_B - p_\pi$ is the four-momentum transfer and $m_B = 5279.4\,\text{MeV}$ and $m_\pi = 139.57\,\text{MeV}$ are the B^0 and π^- masses. For massless leptons, the total decay width is given by

$$\Gamma(B^0 \to \pi^- l^+ \nu_l) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \int_0^\infty \mathrm{d}q^2 [\lambda(q^2)]^{\frac{3}{2}} |f^+(q^2)|^2 \quad (2)$$

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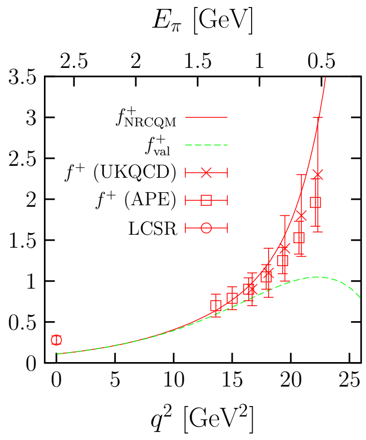

Fig. 1. f^+ form factor obtained with the valence quark (val) contribution alone and with the valence quark plus B^* contribution (NRCQM). We also plot lattice QCD results by the UKQCD [1] and APE [2] Collaborations, and LCSR [3] f^+ results.

with $q^2_{\rm max} = (m_B - m_\pi)^2$, $G_F = 1.16637 \times 10^{-5}\,{\rm GeV}^{-2}$ and $\lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2 = 4m_B^2 |\boldsymbol{p}_\pi|^2$, with $\boldsymbol{p}_\pi$ the pion three-momentum in the B rest frame.

2.1 Nonrelativistic constituent-quark model: valence quark and B*-resonance contributions

Figure 1 shows how the naive NRCQM valence quark description of the f^+ form factor fails in the whole q^2 range (see ref. [4] for details on the calculation). In the region close to $q^2_{\rm max}$, where a nonrelativistic model should work best, the influence of the B^*-resonance pole is evident. Close to $q^2 = 0$ the pion is ultra relativistic, and thus predictions from a nonrelativistic model are unreliable.

As first pointed out in ref. [5], the effects of the B^*-resonance pole dominate the $B \to \pi l^+ \nu_l$ decay near the zero recoil point ($q^2_{\rm max}$). Those effects must be added coherently as a distinct contribution to the valence result. The hadronic amplitude from the B^*-pole contribution is given by

$$-iT^\mu = -i\hat{g}_{B^*B\pi}(q^2) p_\pi^\nu \left(i\frac{-g^\mu_\nu + q^\mu q_\nu / m^2_{B^*}}{q^2 - m^2_{B^*}} \right) i\sqrt{q^2} \hat{f}_{B^*}(q^2) \quad (3)$$

with $m_{B^*} = 5325\,{\rm MeV}$. $\hat{f}_{B^*}$ and $\hat{g}_{B^*B\pi}$ are, respectively, the off-shell B^* decay constant and off-shell strong $B^*B\pi$ coupling constant. Details on their calculation are given in ref. [4] and references therein. From the above equation one can easily obtain the B^*-pole contribution to f^+ which is given by

$$f^+_{\rm pole}(q^2) = \frac{1}{2}\hat{g}_{B^*B\pi}(q^2) \frac{\sqrt{q^2}\hat{f}_{B^*}(q^2)}{m^2_{B^*} - q^2}\,. \quad (4)$$

The inclusion of the B^*-resonance contribution to the form factor improves the simple valence quark prediction down to q^2 values around $15\,{\rm GeV}^2$. Below that the description is still poor.

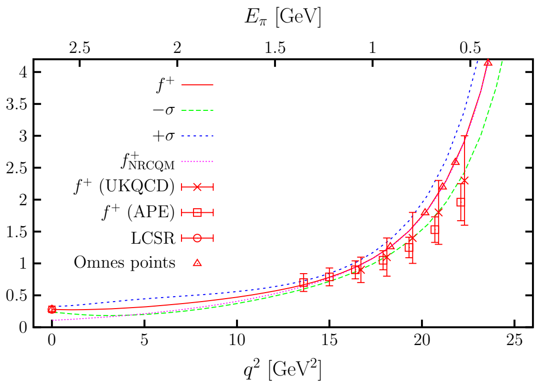

Fig. 2. Omnès improved form factor (solid line). The subtraction points are denoted by triangles. The $\pm\sigma$ lines show the theoretical-uncertainty band.

2.2 Omnès representation

Now one can use the Omnès representation to combine the NRCQM predictions at high q^2 with the LCSR at $q^2 = 0$. This representation requires as an input the elastic $B\pi \to B\pi$ phase shift $\delta(s)$ in the $J^P = 1^-$ and isospin $I = 1/2$ channel, plus the form factor at different q^2 values below the πB threshold where the subtractions will be performed. For a large enough number of subtractions, only the phase shift at or near threshold is needed. In that case one can approximate $\delta(s) \approx \pi$, arriving at the result that

$$f^+(q^2) \approx \frac{1}{s_{\rm th} - q^2} \prod_{j=0}^{n} [f^+(q_j^2)(s_{\rm th} - q_j^2)]^{\alpha_j(q^2)}, \; n \gg 1 \quad (5)$$

with $s_{\rm th} = m_B + m_\pi$ and $\alpha_j(q^2) = \prod_{j\neq k=0} \frac{q^2 - q_k^2}{q_j^2 - q_k^2}$.

Figure 2 shows with a solid line the form factor obtained using the Omnès representation with six subtraction points: we take five q^2 values between $18\,{\rm GeV}^2$ and $q^2_{\rm max}$ for which we use the f^+ NRCQM predictions (valence + B^* pole), plus the LCSR prediction at $q^2 = 0$. The $\pm\sigma$ lines enclose a 68% confidence level region that we have obtained from an estimation of the theoretical uncertainties. The latter have two origins: i) uncertainties in the quark-antiquark nonrelativistic interaction and ii) uncertainties on the product $g_{B^*B\pi} f_{B^*}$, and on the input to the multiply subtracted Omnès representation. See ref. [4] for details.

By comparison with the world average (w.a.) value for the decay width by the Heavy Flavor Averaging Group (HFAG) [6], we obtain

$$|V_{ub}| = (3.4 \pm 0.2({\rm exp.}) \pm 0.7({\rm theo.}))\ 10^{-3} \quad (6)$$

to be compared to the exclusive and inclusive w.a. [6]

$$\begin{aligned} |V_{ub}| &= (3.80 \pm 0.27 \pm 0.47)\ 10^{-3} \quad \text{exclusive w.a.}\,, \\ |V_{ub}| &= (4.39 \pm 0.19 \pm 0.27)\ 10^{-3} \quad \text{inclusive w.a.} \end{aligned} \quad (7)$$

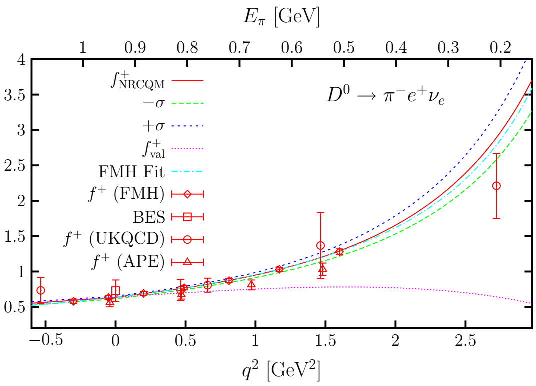

Fig. 3. The solid line denotes our determination of the f^+ form factor ($f^+_{\rm NRCQM}$) for the $D^0 \to \pi^- e^+ \nu_e$ decay. The $\pm\sigma$ lines denote the theoretical-uncertainty band on the form factor. We compare with experimental data by the BES Collaboration [7] and with lattice results by the Fermilab-MILC-HPQCD [8], UKQCD [9] and APE [2] Collaborations.

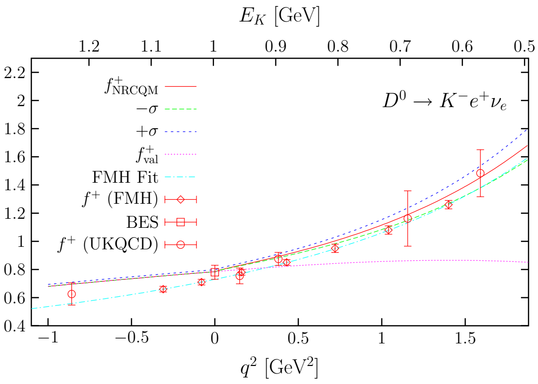

Fig. 4. Same as fig. 3 for the decay $D^0 \to K^- e^+ \nu_e$

3 D $\to \pi l \bar{\nu}_l$ and D $\to K l \bar{\nu}_l$

Our results for the f^+ form factor are depicted in figs. 3 and 4. As before we have considered valence quark plus resonant pole contributions (D^* and D_s^*, respectively). In both cases, we obtain a good description in the physical region of the experimental data [7] and previous lattice results [8,9,2], without using the Omnès dispersion relation. In the case of the $D \to K$ decay, our predictions for negative q^2 values could have been improved by the Omnès representation.

In fig. 5 we compare our results for the $f^+(q^2)/f^+(0)$ with experimental results by the FOCUS Collaboration [10]. We find very good agreement with the data.

Besides we have found for the decay widths

$$\begin{aligned} \Gamma(D^0 \to \pi^- e^+ \nu_e) &= (5.2 \pm 0.1(\text{exp.}) \pm 0.5(\text{theo.})) \\ &\quad \times 10^{-12}\,\text{MeV}, \\ \Gamma(D^0 \to K^- e^+ \nu_e) &= (66 \pm 3(\text{theo.})) \times 10^{-12}\,\text{MeV}. \end{aligned} \tag{8}$$

For $D \to \pi$ we are in good agreement with experimental data while for $D \to K$ our result is two standard deviations higher.

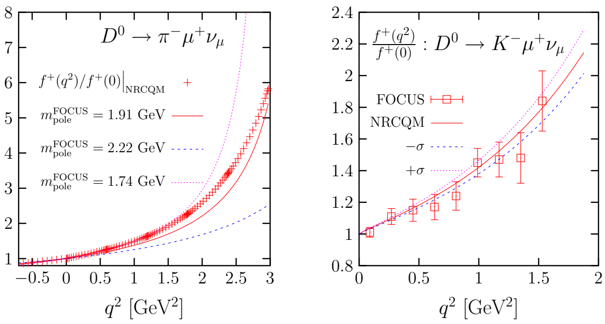

Fig. 5. NRCQM predictions for the ratio $f^+(q^2)/f^+(0)$ for $D \to \pi$ and $D \to K$ decays. We compare with experimental results by the FOCUS Collaboration [10] (a pole fit ($m_{\rm pole} = 1.91^{+0.31}_{-0.17}$ GeV) to data in the $D \to \pi$ case). For the $D \to K$ case we show the theoretical-uncertainty band.

4 Concluding remarks

We have shown the limitations of a pure valence quark model to describe the $B \to \pi$, $D \to \pi$ and $D \to K$ semileptonic decays. As a first correction, we have included vector resonance pole contributions which dominate the relevant f^+ form factor at high-q^2 transfers. Subsequently, for the $B \to \pi$ decay, we have applied a multiply subtracted Omnès dispersion relation. This has allowed us to extend the results of the NRCQM model to the whole q^2 range. Our result for $|V_{ub}|$ agrees within errors with the exclusive and inclusive w.a. by HFAG [6]. For $f^+(q^2)$ of the $D \to \pi$ and $D \to K$ decays and q^2 in the physical region we have found good agreement with experimental and lattice data.

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